

Here are the definitions and conversions of various quantities:

The scattered pressure at the farfield can be defined as

$$p_{scat} = p_0 \frac{e^{ikr}}{r} f_{scat}(ka, \theta) = p_0 e^{ikr} \frac{a}{2r} f_{\infty}(ka, \theta) \quad (1)$$

where  $f_{\infty}$  is the form function and  $f_{scat}$  is the scattering amplitude. The radius of the sphere is  $a$ . The relation between the form function and the scattering amplitude is:

$$f_{scat}(ka, \theta) = \frac{a}{2} f_{\infty}(ka, \theta) \quad (2)$$

Note that the form function is dimensionless and the scattering function has a unit of length. The form function of the backscattering for a rigid & fixed sphere is

$$f_{\infty}(ka, \pi) = \frac{2}{ka} \sum_{n=0}^{\infty} (-1)^n (2n+1) \frac{j'_n(ka)}{h'_n(ka)} \quad (3)$$

The asymptotic value of the form function is unity, i.e.  $f_{\infty}(ka, \pi) \xrightarrow{ka \rightarrow \infty} 1$ . The reduced scattering amplitude is the scattering amplitude normalized by the radius of sphere,  $a$ , and is also a dimensionless quantity:

$$f_{reduced\_scat}(ka, \pi) = \frac{f_{scat}(ka, \pi)}{a} = \frac{1}{2} f_{\infty}(ka, \pi) \quad (4)$$

The differential backscattering cross section is defined as:

$$\sigma_{bs}(ka) = |f_{scat}(ka, \pi)|^2 = \frac{a^2}{4} |f_{\infty}(ka, \pi)|^2 = a^2 |f_{reduced\_scat}(ka, \pi)|^2 \quad (5)$$

The target strength (TS) is

$$TS = 10 \log_{10}(\sigma_{bs}(ka) / A), \text{ with } A = 1 \text{ m}^2 \quad (6)$$

Hence, the relation between the TS and the scattered pressure is

$$\begin{aligned} TS &= 20 \log_{10} \left| \frac{p_{scat}}{p_0} r \right| = 20 \log_{10} |a f_{reduced\_scat}(ka, \pi)| \\ &= 20 \log_{10} |f_{reduced\_scat}(ka, \pi)| + 20 \log_{10} a \\ &= RTS + 20 \log_{10} a \end{aligned} \quad (7)$$

In other words, to convert the normalized scattering amplitude to TS, you need to add  $20 \log_{10} a = 20 \log_{10} 0.25 = -12$  dB. The level of the normalized TS, or RTS, is about -6 dB, so the absolute TS based on the RTS is -18 dB.

Also, from (5), the theoretically asymptotic TS of a rigid sphere is

$$TS(\infty) = 10 \log_{10} \sigma_{bs}(\infty) = 10 \log_{10} \left( \frac{a^2}{4} \right) = 10 \log_{10} \left( \frac{0.25^2}{4} \right) = -18.06 \text{ dB} \quad (8)$$

In summary, if the output from COMSOL is the scattered pressure, i.e.  $p_{scat}$ , I think there is something wrong with the result from COMSOL Multiphysics.